

Topic :
Integration

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MATHEMATICS
CLASS - XII

TIME- 120 MIN

MAXIMUM MARK- 50

General Instructions :

1. This Question paper contains - five sections A, B, C and D. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 15 MCQ's of 1 mark each.
3. Section B has 3 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 3 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

SECTION A

(Multiple Choice Questions)

Each question carries 1 mark

1. The value of the integral $\int \frac{dx}{1 + \tan x}$ is

(a) $\frac{x}{2} + \frac{1}{2} \log |\sin x + \cos x| + c$

(b) $\frac{x}{2} - \frac{1}{2} \log |\sin x + \cos x| + c$

(c) $\frac{1}{2} \log |\sin x + \cos x| + c$

(d) $\frac{1}{2} \log |\sin x - \cos x| + c$

2. $\int x^2 e^{x^3} dx$ equals

(a) $\frac{1}{3} e^{x^3} + C$

(b) $\frac{1}{3} e^{x^4} + C$

(c) $\frac{1}{2} e^{x^3} + C$

(d) $\frac{1}{2} e^{x^2} + C$

3. $\int 2^{2^{2x}} \cdot 2^{2x} \cdot 2^x dx$ is equal to

(a) $\frac{1}{(\log_e 2)} \cdot 2^{2^{2x}} + c$

(b) $\frac{1}{(\log_e 2)^2} \cdot 2^{2^{2x}} + c$

(c) $\frac{2^{2^{2x}}}{(\log_e 2)^3} + c$

(d) $\frac{1}{(\log_e 2)^3} \cdot 2^{2^x} + c$

4. $\int \frac{e^x (1+x)}{\cos^2(xe^x)} dx$ is equal to

(a) $\tan(xe^x) + c$

(b) $\cot(xe^x) + c$

(c) $\cot(e^x) + c$

(d) $\tan[e^x (1+x)] + c$

5. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$ is equal to

(A) $\frac{e^x}{x} + c$

(B) $-\frac{e^x}{x} + c$

(C) $\frac{e^x}{x^2} + c$

(D) $-\frac{e^x}{x^2} + c$

6. $\int e^{5 \log x} dx$ is equal to

(a) $\frac{x^5}{5} + C$

(b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$

(d) $6x^5 + C$

7. $\int_0^{\pi/2} \frac{\sin^2 x dx}{\sin x + \cos x}$ is equal to

(a) $-\frac{1}{\sqrt{2}} \log(\sqrt{2}-1)$

(b) $\frac{1}{\sqrt{2}} \log(\sqrt{2}-1)$

(c) 0

(d) $\log(\sqrt{2}-1)$

8. $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$ is equal to

(A) $7/2$

(B) 7

(C) 14

(D) $15/2$

9. $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$ is equal to

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{12}$

(C) $\frac{\pi}{36}$

(D) $\frac{\pi}{24}$

10. The value of $\int_0^8 |x-5| dx$ is

(A) $15/2$

(B) 9

(C) 7

(D) $9/2$

11. $\int_0^{2/3} \frac{dx}{4+9x^2}$ is equal to

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$

(D) $\frac{\pi}{4}$

12. $\int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ is equal to

(a) 1

(b) 2

(c) 3

(d) 4

13. $\int \frac{x^3}{x+1}$ is equal to

(a) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log |1-x| + C$

(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log |1-x| + C$

(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log |1+x| + C$

(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log |1+x| + C$

14. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

(a) $2(\sin x + x \cos \theta) + C$

(b) $2(\sin x - x \cos \theta) + C$

(c) $2(\sin x + 2x \cos \theta) + C$

(d) $2(\sin x - 2x \cos \theta) + C$

15. $\int \frac{\sin x - x \cos x}{x(x + \sin x)} dx$ is equal to

(a) $\log \left| \frac{x}{x + \cos x} \right| + c$

(b) $\log \left| \frac{x}{x + \sin x} \right| + x + c$

(c) $x - \log \left| \frac{x}{x + \sin x} \right| + c$

(d) $\log \left| \frac{x}{x + \sin x} \right| + c$

SECTION B (Answer any THREE)

This section comprises of very short answer type-questions (VSA) of 2 marks each

16. Evaluate $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$.

17. Evaluate $\int \frac{1}{e^x+1} dx$.

18. Evaluate $\int \sqrt{(x-3)(5-x)} dx$

19. Evaluate $\int \frac{dx}{1+\cos x}$

20. Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

21. Evaluate $\int \frac{dx}{1+\tan x}$

SECTION C (Answer any THREE)

(This section comprises of short answer type questions (SA) of 3 marks each)

22. Prove that $\int_0^{\pi/4} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$

23. Evaluate $\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$

24. Evaluate $\int_1^2 \frac{xdx}{(x+1)(x+2)}$

25. Evaluate $\int_1^2 \frac{5x^2}{x^2+2x+3} dx$

26. Evaluate $\int_0^1 \frac{dx}{e^x + e^{-x}}$

SECTION D (Answer any FOUR)

(This section comprises of long answer-type questions (LA) of 5 marks each)

27. Evaluate $\int \frac{xdx}{(x+1)^2(x+2)}$

28. Evaluate $\int_0^\pi \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x}$

29. Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$

30. If $\frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then find the value of a and b ?

31. Evaluate $\int e^{\tan^{-1}x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$

32. Evaluate $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

“Wishing you a calm mind and steady hand as you take your exam.”